University of California at Berkeley
Physics 129A
Professor Freedman
Fall 2004
September 2, 2004
Homework #1 (Due: Friday September 10)

1. Show that when non-relativistic neutrons of kinetic energy E_o collide with nuclei with mass number A at rest, the smallest energy that the elastically scattered neutrons can have is given approximately by

$$E_{\min} = E_o \left(\frac{A-1}{A+1} \right)^2 .$$

What will be the approximate energy of the neutrons after one, two and any number (n) of head-on collisions, if the target is hydrogen, carbon, and iron? In a more realist situation you must consider that the collisions will not all be head on and the finite temperature of the scatterer has an effect. In this case, however, make a simple estimate the final neutron energy (or at least the range of energies) after very many collisions. Will this final energy of the neutrons be different for the different scattering materials? (If you are ambitious you can work out the more general case for scattering at an angle θ in the laboratory, but this is not being required of you right now.)

Enrico Fermi was the first one to consider this important problem. The result is essential for understanding how to build nuclear reactors and for doing experiments with low-energy neutrons. To get the formula you should approximate the mass of the neutron and proton as equal and neglect the nuclear binding for the nuclei involved.

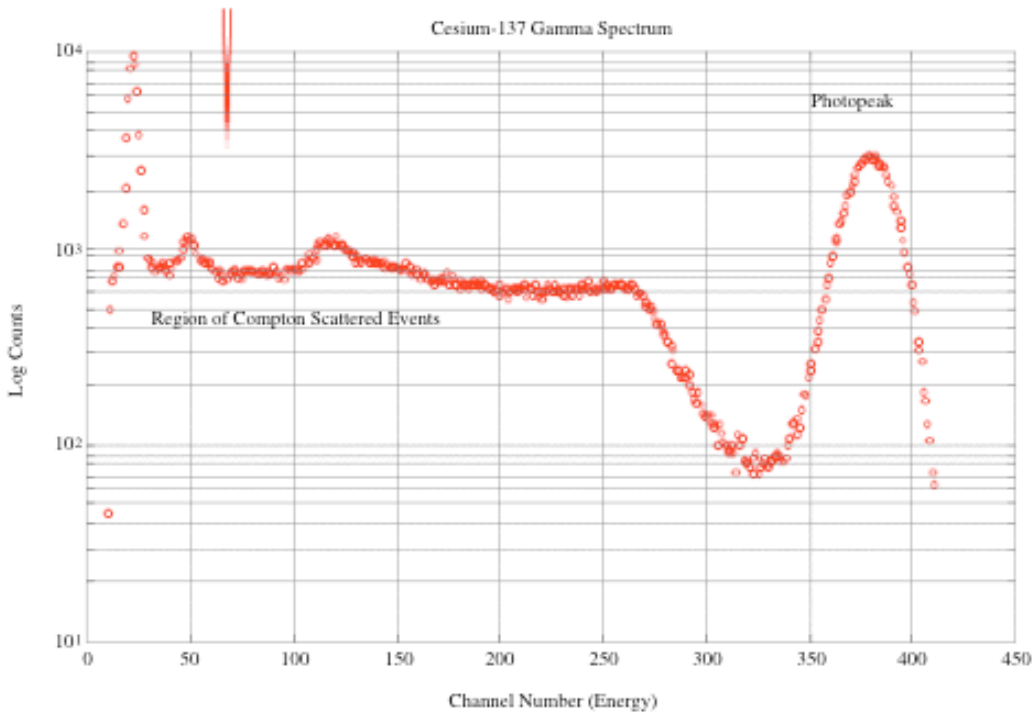
2. (After Problem 1.3 in Griffiths) Before the discovery of the neutron most people believed that nuclei were made up of protons and electrons (the only two particles known at the time). Using the uncertainty relationship for momentum and position estimate the minimum energy spread of the electron within a typical nucleus of radius (say one or two Fermis). Discuss whether or not an electron could possibly bound by the ordinary electromagnetic interaction to this size orbit. Could there be another force of the kind required without messing up the structure or behavior of atoms? Following Griffith's suggestion, compare the electron energy with the energy of the electron emitted in tritium beta decay. Is there an inconsistency? There is another problem with the nucleus consisting of protons and electrons. Both ^{6}Li and ^{14}N nuclei are known experimentally to be bosons (because they both have spin 1 in their ground states). Show that this conclusion is inconsistent with the idea of nuclei being made of protons and electrons.

3. (After Problem 3.4 in Griffiths) Cosmic-ray muons are produced at the top of the atmosphere (you can take Griffiths' estimate, 8,000 m, although a larger number is more correct) and travel at very nearly the speed of light (0.998 c, say)

(a) The muon is lifetime is about 2.2 x 10^{-6} sec. Show that a naïve calculation that neglects relativity would imply that the muon decays before reaching the Earth's surface.
(b) Show that including the effect of relativistic time dilation changes the conclusion from part a.
(c) Now analyze the same problem in the rest frame of the muon where the lifetime really is 2.2 μsec. Explain why these results are consistent.
(d) As Griffiths suggests the muons do come from the decay of positively and negatively charged pions produced primarily by the interactions of cosmic-ray protons with Nitrogen and Oxygen molecules. Look up the lifetime of the charged pions and determine whether or not they have much of a chance of reaching the ground if their velocity is 0.998 c. What is the flux of cosmic rays at the Earth's surface and what is the primary component? You will have to look this up somewhere unless you are very good at doing experiments. I suggest you take a look at the Particle Data Group web site (http://pdg.lbl.gov/); look for the section on "cosmic rays". It would be a good idea for you to get acquainted with this site since it has lots of very interesting information about particle physics.

4. A photon of wavelength λ collides elastically with an electron of mass m. If the photon scatters at angle θ, find its outgoing wavelength. This is the famous Compton formula. Derive an expression for the energy of the recoil electron from an incident photon of a given energy and outgoing scattering angle. This exercise should help you with the rest of the problem. (e.g. this is a hint.)
The figure below is the spectrum of photons from Cesium-137 decay as detected in a NaI detector. If you never heard of this kind of detector then you should look it up on the web. You can look up the decay scheme (you can find it in many books but a good place to look is (http://ie.lbl.gov/toi.html). You will find that ^{137}Cs beta decays to ^{137}Ba and that the dominant radiation is a 661.6 keV gamma ray. A 30.7 keV X-ray is also emitted. Can you guess why? Use your equation from the first part of this problem to help understand the experimental spectrum shown in the figure. The "photopeak" should correspond to 661.6 keV photons that are totally absorbed in the crystal. We want you to explain the feature below the photopeak between channels 275 and 325. The solution we might suspect is that this feature arises from the simple Compton scattering process. Does this explain the feature? Remember that the horizontal scale is in some arbitrary

channel number scale not in physical energy. You will have to make a conversion.



5. Since you are having such a good time surfing the web here is an easy problem that will require you to find out the masses of some nuclear isotopes.
(a) Calculate the approximate density of nuclear matter in gm/cm^3.
(b) Calculate the difference between the binding energy of a nucleus of ^{12}C and the sum of three ^{4}He nuclei (α particles). Assuming that ^{12}C is composed of three α-particles in a triangular structure, with three effective "α-bonds" between them, what would be the binding energy per α-bond?